Sketching as a Spatial Tool: A Qualitative Study of Grade Three Students' Representation of Reflection

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The concept of symmetry is intrinsic to spatial reasoning and mathematics. Yet many students struggle with the concept, as well as its wide-reaching affordances for mathematics. Through a study of students' drawings of reflection we were able to identify some characteristics of students' representation of bilateral symmetry, and where errors commonly occur. We propose suggestions for how these findings can be linked more broadly to mathematics instruction.

Spatial reasoning is considered a powerful tool in the teaching and learning of mathematics (Ishikawa & Newcombe, 2021). Countries around the world have incorporated spatial reasoning into core curriculum standards (e.g., Canada, Singapore) and the notion of space forms a content strand of the new Australian curriculum (ACARA, 2022a). At an individual skill level, spatial visualisation is deemed crucial to supporting mathematics achievement (Hawes et al., 2022). However, much of what we know about spatial visualisation has emerged from psychological studies based on testing (Lowrie et al., 2020), or inferred when a task is considered spatial (Patahuddin et al., 2020). Furthermore, spatial visualisation is often used as a catch-all phrase for complex mental manoeuvres with little regard for how this skill is implemented in mathematical practice (Ramful et al., 2015). This study was designed in response to the high level of interest in spatial visualisation for supporting mathematics understanding, and the need for further research on the nature of spatial reasoning beyond performance on spatial tests (Ishikawa & Newcombe, 2021).

One component of spatial visualisation that is contained both within the psychological construct (Linn & Petersen, 1985; Lowrie & Logan, 2018) and as a central idea in mathematics (ACARA, 2022a; Ng & Sinclair, 2015) is symmetry. Symmetry is often discussed in terms of folding over a line (Leikin et al., 2000), and in fact, that is how the construct is often measured (i.e., Paper Folding Test; Ekstrom et al., 1976). However, focusing on the line of symmetry has been shown to impact the ability to recognise symmetry in different ways (Clements & Battista, 1992; Mulligan et al., 2020). In this study, we analysed student drawings of reflections, with a focus on spatial representation of symmetry as a proposed precursor to mathematical language.

Theoretical Framing of Spatial Visualisation

Spatial visualisation (SV) is a spatial skill, distinct from visualisation (Vs). The former (SV) is primarily a psychological construct used to predict or train skills (Hawes et al., 2022), and is often operationalised by tests requiring complex, multistep spatial manoeuvres, rather than a clearly defined skill (Linn & Petersen, 1985). By contrast, visualization (Vs) is a term more dominant in mathematics education and concerns the creation and interpretation of visual representations, both in the mind's eye and in concrete form (Arcavi, 2003; Gutiérrez, 1996). These two constructs are woven together in work by Hegarty and Kozhevnikov (1999) and Presmeg (1986), where the ability to visualise spatially was closely related to performance on mathematics tasks, while the ability to visualise tended to result in representations that were lacking the critical information to support successful problem-solving. In fact, Kozhevnikov et al. (2010) found a trade-off between one's

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ability to visualize objects and visualize spatially. The magnitude of this difference increases with age, where as one type of skill develops, the other often wanes, particularly in the science and visual arts fields.

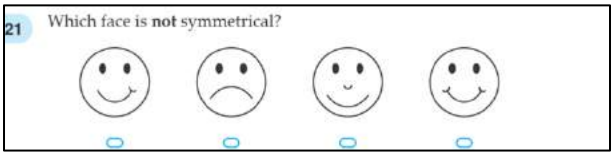
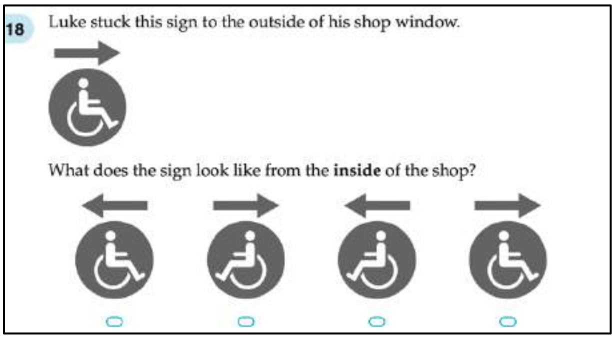
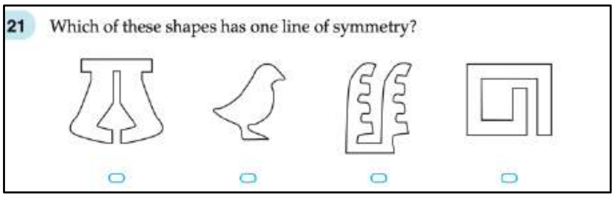
Symmetry/Reflection

In this paper we are focused on spatial visualisation in the form of primary students' ability to represent bilateral symmetry through reflections. The new Australian mathematics curriculum introduces symmetry in year 4 (as opposed to year 3 in the previous curriculum) through identifying environmental symmetry, line and rotational symmetry, and creating symmetrical patterns (ACARA, 2022a). It is noteworthy that this curriculum includes symmetry in movement and shape from year 3 within Health and Physical Education with links to mathematics learning areas. This reflects the idea that symmetry has implications and opportunities for development across a range of areas.

Students tend to struggle with the concept of symmetry (Clements & Battista, 1992; Sarama & Clements, 2008). Mulligan et al. (2020) found that a common error amongst students results from focusing on lines of symmetry of the bounding shape, not the spatial relations within. It is possible that students are struggling more with the mathematical language around symmetry than lacking spatial competence. To examine this, we consider performance on three publicly available symmetry items from the National Assessment Program—Literacy and Numeracy (NAPLAN; ACARA, 2022b) as a means of understanding broad understanding and challenges surrounding symmetry comprehension (see Table 1).

Table 1

*NAPLAN Year 3 Symmetry Items (*ACARA, *2022b)*

Item		Year	N	Success	Errors
1		2013	9847	A: 50%	B: 34% C: 11% D: 4%
2		2012	3828	C: 59%	A: 7% B: 8% D: 26%
3		2012	3799	A: 54%	B: 14% C: 11% D: 20%

In Table 1 success rates are above chance, but the errors provide insights into student difficulties. For item 1, half the students had both the conceptual knowledge and ability to discriminate between symmetrical and non-symmetrical images. Of those that were incorrect, the largest proportion chose the image depicting a negative emotion, perhaps reflecting the negative wording of the question.

Item 2 measured students' ability to visualise and could be completed using a perspective-taking strategy. The greatest proportion of errors indicate spatial difficulties, but the item had the highest success rate, coinciding with no language around symmetry. For Item 3, just over half the students identified a line of symmetry but the distribution across the alternate responses suggests that some students had trouble with the question demands. This may indicate a lack of exposure to explicit mathematical language such as the term *symmetry*, rather than students' spatial knowledge of the concept of symmetrical.

Some have argued that young children's capacity for understanding symmetry is much greater than curriculum guidelines (and assessment) would suggest, and that early introduction to symmetry concepts, through engagement with concrete materials, can provide a strong foundation for later problem-solving (Ng & Sinclair, 2015; Sarama & Clements, 2008). Through intentional teaching, exposure to spatial tasks helps students build understanding of spatial transformations like rotation and reflection (Mulligan et al., 2020).

Sketching Representations in Mathematics

Visualisation, in the form of pictorial representation, is an important component of mathematics (Arcavi, 2003; Gutiérrez, 1996; Way, 2021). Presmeg and Balderas-Cañas (2001) cite two purposes of visualisation in mathematics: to make sense and to solve. For both functions, technical drawing knowledge and drawing quality are not the most critical elements, rather an ability to extract and represent key information (Rellensmann et al., 2021). The language used to differentiate realistic depictions from simplistic representations that embody critical relations varies between studies (e.g., Hegarty & Kozhevnikov, 1999; Presmeg, 1986), but these abstractions support the shift from visual to symbolic representation (Lowrie, 2020). However, incorporating drawing into mathematical representation is not a natural phenomenon, even amongst young children, and therefore may require teacher help to incorporate drawing into mathematical practice (Bakar at al., 2016).

Context of this Study

Children can differentiate between symmetrical and asymmetrical; however, 1) this is a static process involving visual discrimination, not an understand of the relationships between the components (Ng & Sinclair, 2015); and 2) in advanced mathematics, even after years of instruction, many still struggle to identify lines of symmetry (Clements & Battista, 1992). Therefore, although young children can understand the concept of symmetry, the mathematical language can be a hurdle to demonstrating this knowledge. In this study we focus on student drawings as a means of understanding their spatial representations (Lowrie & Logan, 2018; Way, 2021). We sought to combine two related, yet previously isolated, lines of enquiry:

- SV instruction is beneficial for mathematics outcomes (Hawes et al., 2022). Yet much of the intervention work remains focused on spatial skills training, instead of activities aligned to mathematics that promoting spatial reasoning (Lowrie et al., 2020).
- Sketching is helpful for understanding students' mental processes (Way, 2021) and supports learning (Bakar et al., 2016). However, the way students demonstrate their spatial visualization skills using pictorial representations has not been examined.

Research questions. We focus on students' sketched reflections to extend what we know about SV from studies based on testing. Although young children can perform symmetry tasks earlier than curriculum suggests, they may not have the language or capability to express their spatial thinking. Here we present an intentional spatial lesson designed to help students think spatially and apply their SV skills through demonstration of reflection to answer the questions:

- How do grade 3 students use self-generated drawings to encode their SV skills?
- What impact does type of representation (i.e., detailed or structured) have on accuracy?

Method

Participants and Method

The participants were Grade 3 students from three NSW primary schools. Students were given a series of photographs to reflect, or the option to generate their own. The photographs were images of children participating in physical activities (see examples in Figure 1).

Figure 1. Sample images provided to students for reflection activity.

Students were provided with mirrors, drawing implements, and paper or whiteboards. The open nature of the task meant that students were free to complete the task as they wished with minimal constraints. Drawings were photographed for later analysis. Here we present a sample of data from students who chose a soccer player ($N = 8$), ballerina ($N = 10$), or free drawn houses ($N = 6$). Ethics approval was granted by the authors' university and relevant state educational jurisdictions. Parental consent was sought from all students before participation.

Data Analysis

We coded student drawings using the constant comparison methodology reported by Corbin and Strauss (2008). In this way, we drew meaning and classifications from the drawings through similarities, differences, and errors. student drawings were categorised as detailed (i.e., containing multiple/complex elements that add to visual detail but are extraneous to subject structure) or structured (i.e., containing key structural components only e.g., stick figures or minimal geometric shapes). The coding rubric we used for errors is presented in Table 2.

Table 2

Classification of Student Errors

Errors	Coding/Description
E1	Failure to reflect large components (e.g., legs, arms)
E2	Failure to reflect small components (e.g., facial features)
E3	Direction of large components (e.g., body—limbs pointing the wrong way would be E1)
E4	Direction of small components (e.g., hair, patterns)
E5	Missing detail in reflection (e.g., patterns or objects)
E6	Errors in spatial relations between components

Although proportionality is highly studied in student drawings (Quane et al., 2021), the focus in our coding was on the representation of attributes and spatial relations in line with our conceptual distinction between Vs and SV.

Results

Just over half of the drawings contained a line of symmetry (14/24). Most students chose to recreate the photos as well as the reflected version (16/18). This required students to decode the photo and extract critical information, this representation became the source of their reflection. Student recreations of the photos were the basis of our analysis, not the accuracy relative to the original. In terms of errors in the reflected drawings, we categorised these based on the subject and the nature of the depiction (i.e., detailed versus structured; see Table 3).

Table 3

Error Types and Frequencies for Detailed and Structured Drawings

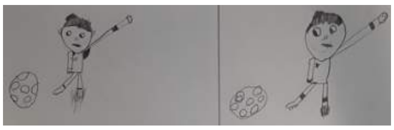
	Detailed							Structural						
Subject	N	E1	E2	E3	E4	E5	E6	N	E1	E2	E3	E4	E5	E6
Ballerina	7	3	1	2	1	4	2	3	1	2	1	1	5	1
Soccer	2	1	2	1	2	1	1	6	0	1	0	0	0	2
House*	4	0	2	0	0	1	2	2	0	0	0	0	0	0

*Freeform—no photo provided.

The ballerina drawings included a higher proportion of detailed drawings compared with the soccer drawings. This difference is not merely a result of the photo content, as both images include details on the clothing and shoes of the subjects as well as the unusual body and face orientation. We present examples of detailed and structured drawings for each of the subject categories with varying degrees of error (see Table 4). These drawings were typical of other representations that we observed.

Table 4

Sample Drawings in the Different Categories

	Ballerina	Soccer	House
Detailed	 Some elements reflected such as the direction of the face and the position of the bun. However, the legs are translated rather than reflected (E1).	 Explicit detail about boy and ball, but little indication that the student understood the term symmetry as both images are positioned the same way (E1, E2).	 Main form is reflected across line of symmetry but components (i.e., windows) not reflected in terms of number or position (E5, E6)
Structured	 Subject structure reflected to match original but missing details (E6) and small errors (E2, E4).	 Boy and soccer ball show minimal detail and fairly accurate reflected images, down to the grass.	 Another structural drawing with minimal detail but well-matched reflected components.

From these drawings it is apparent that the detailed drawings did leave more opportunities for error, on both large (E1; see Ballerina legs in the detailed drawing) and small (E4; see Ballerina hair in detailed drawing) scales. By contrast, the detailed house drawing errors presented in Table 3 demonstrate primarily missing details in the reflected images compared with the structural drawings which contained no errors.

Discussion

In this paper we explored student representation of bilateral symmetry and found that even at a young age, students were able to accurately generate reflected drawings of complex images. The use of detailed or structured drawings was associated with the subject, and the nature of the errors varied by subject as well. That is, geometric drawings of houses and the representations of a soccer player which used a structured approach contained fewer errors. By contrast, the types of errors varied in proportion between detailed and structured representations for the ballerina drawings, both contained missing parts, but more large components were not reflected in the detailed drawings, compared with small components in the structured drawings. It is noteworthy that even the structured ballerina drawings contained a greater amount of detail on the central subject than the soccer drawings (see Table 4).

The accuracy with which reflections were depicted for structured drawings is consistent with existing work in mathematics where representations focused on key structural relations were less error-prone than detailed images (Bakar et al., 2016; Hegarty & Kozhevnikov, 1999). However, as there was no mathematical content to extract in this task it is interesting that this pattern still holds. The drawings generated by students to represent symmetry provide insights into their internal spatial representations (Way, 2021). Even in the absence of mathematical content, similar patterns emerged in students' representations in terms of their use of detail or structure to represent symmetry. For most students, some measure of reflection was represented, indicating they understood the concept of symmetry. However, the nature of the encoded representations, the subject matter, and their use of the line of symmetry impacted their ability to demonstrate this knowledge.

Efficient Encoding

There were a smaller proportion of errors for the house drawings, compared with the photos, in fact none for the structured house drawings contained errors. There are two possibilities for this: 1) the geometric structure of the houses made it easier for one-to-one encoding, or 2) there was no need to decode an original image, thereby reducing the amount of cognitive work required in the task. However, it is less likely to be (2) as most students generated their own images for reflection even for the photos. Therefore, when supporting students to use drawings the content matters in how accurately they can encode the representation. This aligns with Lowrie's (2020) analysis of a word problem requiring analysis of an array of chairs, the subject was simple, but the more components students included in their drawings, the more opportunity there was for error in solving the task.

Line of Reflection

Just over half the students chose to include a line of symmetry, however, even in the absence of the line students demonstrated their ability to conceptualise bilateral symmetry. Despite the provision of tools such as mirrors, it seems that students treated each drawing as independent and the line of symmetry between each side was not a defining characteristic of the drawing. This has implications for how we conceptualise symmetry. Much of the curriculum-related symmetry content exists within bounded geometric objects (ACARA, 2022a; Mulligan et al., 2020). However, symmetry exists in many forms within mathematics and the world more broadly (Ng & Sinclair, 2015), as reflected by the new Australian curriculum (ACARA, 2022). Therefore students' conceptualisation of symmetry and the classroom learning around the construct needs to be broader too.

Implications of our Study for Teaching and Learning

At a surface level this task was about students' ways of representing reflection, however the rich, underlying mathematical opportunities in a task such as this are important for future work (Ng & Sinclair, 2015). Symmetry tasks promote relational thinking in terms of spatial organisation that have implications when understanding the construction of coordinated reference systems such as the cartesian plane (Clements & Battista, 1992).

Reflection and symmetry are important for higher level mathematics (Clements & Battista, 1992; Leikin et al., 2000; Ng & Sinclair, 2015). For example, in geometry, lines of symmetry in geometrical figures are explicit. However, there is also an opportunity to use symmetry to help students with number-based problems. Consider the balance between sides of an equal sign, rather than a one-directional operation, reframing conceptual understanding to include the notion of equivalence may help students with more complex mathematical content (Kieran, 1981; Patahuddin et al., 2020). Currently the Australian Curriculum refers to symmetry in dynamic terms such as flip or fold. If other language was incorporated to reflect balance and equivalence, not just the end result of an operation, we may see better transfer to number-based work providing foundations for algebraic reasoning.

Limitations and Future Directions

Many students naturally use graphical representations in their play, although there is still some reluctance in mathematics problem-solving (Bakar et al., 2016). If students are supported to encode representations using structural relations they will have greater opportunities for success. However, it is evident from this and other work (Hegarty & Kozhevnikov, 1999) that some students don't naturally identify the critical components and therefore errors in representations occur. This has implications for future work in that 1) students need support to focus on key structural components for spatial representations, and 2) effective spatial representations need to be linked to mathematical content in a way that builds conceptual understanding. We did not connect students' spatial representations with their mathematical competence in this study, that is an avenue for future work.

There is a possibility that gender had an impact on the choice of subject matter and the use of detailed or structured representations. We did not collect gender information in this lesson, but future work could explore the role of gender in the use of graphic representations.

Conclusion

Students represent space and relations in different ways and although we know these skills are important, we need to 1) foster their development in the context of mathematics classrooms, and 2) not reduce children's spatial thinking down to performance on tests. There is a wealth of literature to draw on to build thoughtfully designed spatial tasks that take advantage of students' skills and move beyond the drill and practice of spatial tests to support mathematical understanding.

References

Australian Curriculum, Assessment and Reporting Authority. (2022a). *The Australian curriculum: Mathematics*. ACARA. https://v9.australiancurriculum.edu.au/

Australian Curriculum, Assessment and Reporting Authority. (2022b). *NAPLAN sample tests.* ACARA. https://www.nap.edu.au/nap-sample-assessments

Arcavi, A. (2003). The role of visual representations in the learning of mathematics. *Educational Studies in Mathematics, 52,* 215–241.

Bakar, K. A., Way, J., & Bobis, J. (2016). Young children's drawings in problem solving. In B. White, M. Chinnappan, & S. Trenholm (Eds.), *Opening up mathematics education research. Proceedings of the 39th annual conference of the Mathematics Education Research Group of Australasia* (pp. 86–93). Adelaide: MERGA.

Clements, D. H., & Battista, M. T. (1992). Geometry and spatial reasoning. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 420–464). Macmillan.

Corbin, J., & Strauss, A. (2008). *Basics of qualitative research (3rd ed.): Techniques and procedures for developing grounded theory*. SAGE Publications.
Ekstrom, R. B., French, J. W., Harman, H. H., & Dermen, D. (1976). *Kit of factor-referenced cognitive tests*. Educational Testing Service.
Gutiérrez, A. (1996). Visualization in 3-dimensional geometry: In search of a framework. In L. Puig, & A. Gutiérrez (Eds.), *Proceedings of the 20th international conference for the Psychology of Mathematics Education* (Vol. 1, pp 3–19). Valencia, Spain.
Hawes, Z., Gilligan-Lee, K. A., & Mix, K. S. (2022). Effects of spatial training on mathematics performance: A meta-analysis. *Developmental Psychology, 58*(1), 112–137.
Hegarty, M., & Kozhevnikov, M. (1999). Types of visual-spatial representations and mathematical problem solving. *Journal of Educational Psychology, 91*(4), 684–689.
Ishikawa, T., & Newcombe, N. S. (2021). Why spatial is special in education, learning, and everyday activities. *Cognitive Research: Principles and Implications, 6*(20).
Kieran, C. (1981). Concepts associated with the equality symbol. *Educational Studies in Mathematics, 12*(3), 317–326.
Kozhevnikov, M., Blazhenkova, O., & Becker, M. (2010). Trade-off in object versus spatial visualization abilities: Restriction in the development of visual-processing resources. *Psychonomic Bulletin & Review, 17,* 29–35.
Leikin, R., Berman, A., & Zaslavsky, O. (2000). Learning through teaching: The case of symmetry. *Mathematics Education Research Journal, 12*(1), 18–36.
Linn, M. C., & Petersen, A. C. (1985). Emergence and characterization of sex differences in spatial ability: A meta-analysis. *Child Development, 56,* 1479–1498.
Lowrie, T. (2020). The utility of diagrams in elementary problem-solving. *Cognitive Development, 55*(July-September).
Lowrie, T., & Logan, T. (2018). The interaction between spatial reasoning constructs and mathematics understandings in elementary classrooms. In K. S. Mix, & M. T. Battista (Eds.), *Visualizing mathematics: The role of spatial reasoning in mathematical thought* (pp. 253–276). Springer.
Lowrie, T., Resnick, I., Harris, D., & Logan, T. (2020). In search of the mechanisms that enable transfer from spatial reasoning to mathematics understanding. *Mathematics Education Research Journal, 32*(2), 175–188.
Mulligan, J., Oslington, G., & English, L. (2020). Supporting early mathematical development through a 'pattern and structure' intervention program. *ZDM Mathematics Education, 52,* 663–676.
Ng, O., & Sinclair, N. (2015). Young children reasoning about symmetry in a dynamic geometry environment. *ZDM Mathematics Education, 47*, 421–434.
Patahuddin, S. M., Rokhmah, S., & Ramful, A. (2020). What does teaching of spatial visualisation skills incur: An exploration through the visualise-predict-check heuristic. *Mathematics Education Research Journal, 32*(2), 307–329.
Presmeg, N. (1986). Visualization in high school mathematics. *For the Learning of Mathematics, 6*(3), 42–46.
Presmeg, N., & Balderas-Cañas, P. E. (2001). Visualization and affect in nonroutine problem solving. *Mathematical Thinking and Learning, 3*(4), 289–313.
Quane, K., Chinnappan, M., & Trenholm, S. (2021). Draw yourself doing mathematics: Developing an analytical tool to investigate the nature of young children's attitudes towards mathematics. *Mathematics Education Research Journal.* https://doi.org/10.1007/s13394-021-00399-2
Ramful, A., Ho, S. Y., & Lowrie, T. (2015). Visual and analytical strategies in spatial visualisation: Perspectives from bilateral symmetry and reflection. *Mathematics Education Research Journal, 27*, 443–470.
Rellensmann, J., Schukajlow, S., Blomberg, J., & Leopold, C. (2021). Does strategic knowledge matter? Effects of strategic knowledge about drawing on students' modeling competencies in the domain of geometry. *Mathematical Thinking and Learning*, https://doi.org/10.1080/10986065.2021.2012741
Sarama, J., & Clements, D. H. (2008). Mathematics in early childhood. In O. N. Saracho, & B. Spodek (Eds.), *Contemporary perspectives on mathematics in early childhood education* (pp. 67–94). Age Publishing Inc.
Way, J. (2021). Issues and affordances in studying children's drawings with a mathematical eye. In Y. H. Leong, B. Kaur, B. H. Choy, J. B. W. Yeo, & S. L. Chin (Eds.), *Excellence in mathematics education: Foundations and pathways. Proceedings of the 43rd annual conference of the Mathematics Education Research Group of Australasia* (p. 109). Singapore: MERGA.